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STRATEGIC TIMING AND PRICING OF A SUBSTITUTE
IN A CARTELIZED RESOURCE MARKET

by
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1. Introduction*

Recent price increases by the OPEC cartel and growing concern over the depletion of key resources have prompted consuming nations to consider the development and pricing of substitutes for conventional resources. Although the long term benefits from pursuing an R&D policy are recognized, there is less consensus on the speed at which a development program should proceed.[1] Accelerated development, while setting a lid on prices after the substitute becomes available, may encourage the cartel to raise prices prior to this, and thus partially dissipate the benefits from the R&D program. The incentive to accelerate the program is also related to the increased costs of doing so. Another consideration is the pricing of the substitute. Government policies making the substitute cheaper or more expensive to the consumer may also affect cartel supply of the resource. An understanding of these problems of pricing and of pacing technological development in a cartelized resource market requires a careful analysis of the strategic interactions between the resource cartel and the consuming nations.

Strategic pricing and research and development by consumers has largely been neglected in the exhaustible resources literature. Notable exceptions are the recent papers by Gilbert and Goldman (1978), Hoel (1978), and Salant (1979) that examine a cartel's response to the introduction of an exogenously developed "backstop technology," available at a fixed price. However, these papers do not consider whether immediate development of a substitute is feasible, or desirable when compared with a more gradual development program.

This paper considers a bilateral monopoly in an international market for the resource. A cartel of producing nations is assumed to own all the known resources; the consuming nations have the technical capability for developing a substitute, which can then be produced at a constant marginal cost. The paper focuses on the optimal strategy for pricing and developing the exhaustible resource substitute and its effectiveness in controlling the market power of the resource cartel. In a dynamic game between producers and consumers, the resource extraction program and development time of the substitute are determined.

The form of the game depends upon whether the strategic interaction is likely to be asymmetric, in that one side has an advantage of timing or other means of making credible commitments. In this paper we examine the case where consuming nations have a credible means of pre-commitment in choosing when to introduce the substitute. In essence, the government promises to provide consumers with the substitute at some future date and at a specified price. We assume, that as a practical matter, the agreement is legally binding, and that the promise is made credible by the amount of government expenditure on development of the substitute. The resource cartel, on the other hand, has no such device for pre-commitment. A strategic advantage for the cartel could only be achieved by making binding futures contracts for the sale of all or part of the stock of the resource. However, since future agreements for the sale of internationally traded commodities are difficult to enforce, we assume that the cartel cannot credibly bind itself to following a particular supply path.[2] The consuming nations, therefore, would seem well equipped to play the role of Stackelberg leader, while the cartel responds passively to the former's strategy. The paper focuses on this model.[3]

Our primary attention in this paper is given to the date of introduction of the substitute technology as a strategic instrument of consuming nations. The introduction date is depicted as a determinate function of R&D expenditure committed at the beginning of the planning horizon. The justification for this assumption is that the R&D activity is confined to development projects for which the technology is reasonably well understood; only the feasibility of the process at large scales of production remains to be tested, involving the construction of demonstration plants. Since capital costs predominate at this stage of development, the assumption that total research expenditure as committed at the initial planning date is not unreasonable.[4]

The present value of research costs required for development of the technology decreases as the date of introduction is postponed. Given the determinate cost-time relationship, the amount of investment committed signals to the cartel when the substitute will become available. The consuming nations, acting as a Stackelberg leader, recognize the cartel's reaction to the timing of the substitute in choosing their development strategy. The cartel, on the other hand, has no choice but to take the introduction date of the substitute as given in choosing a profit maximizing extraction path.

The consumer-as-Stackelberg leader model is developed in Section 2. In at least two important respects, the results run counter to conventional scenarios which describe the impact of potential substitutes on resource markets. First, a basic tenet of policy discussion is that substitutes for exhaustible energy resources are to be brought on line as soon as the price of the resource rises to the supply price of the substitute. In the

model presented here, an "over-shooting" equilibrium price path is obtained, in which the resource price is allowed to rise above the substitute supply price for some period of time before the substitute is introduced. At the time the substitute comes on line, price falls discontinuously to the back-stop supply price.[5]

Second, an increase in R&D expenditure in order to hasten the introduction of the substitute may be undesirable. By committing themselves to a reduced development program, the consuming nations can induce the cartel to accelerate extraction of the resource, as well as reduce the present value cost of developing the substitute.

Section 3 considers the possibility that the governments of consuming nations may influence the purchase price for the substitute as a further means to control cartel production. There, we identify instances where decreasing (increasing) the substitute price by subsidizing (taxing) purchases of the substitute can favorably affect cartel production from the consumer's viewpoint.

The final section summarizes the major findings of the paper and suggests some directions for further theoretical development of the underlying model.

2. The Model with Consumers as Stackelberg Leaders

2.1 The Structure of the Model

To focus on the dynamic and strategic interactions between resource producers and consumers, a model is presented which abstracts from complexities such as uncertainties in the R&D process, in demand, and in the size of the resource stock. A producing cartel is assumed to own a finite amount

of an exhaustible resource stock at t=0, S(0), which can be extracted at zero cost. Collectively, consumers have potential access to a technology that can produce a perfect substitute at a constant marginal cost p. We assume that the purchase price of the substitute is equal to $\bar{p}$. This is relaxed in Section 3. Consumers are assumed to commit themselves to a research and development program at the beginning of the planning horizon that guarantees the availability of the backstop at a specified time T. The cartel acts passively, choosing a profit-maximizing path, subject to the consumer's choice of T.

Let

$U(x+y)$ = the utility or benefit function of consuming nations for flows of the resource x and/or the perfect substitute y

$p(x+y)$ = the corresponding inverse demand function

$R(x+y)$ = the corresponding revenue function

$\bar{p}$ = the constant cost of production of the perfect substitute

$\bar{x} = p^{-1}(\bar{p})$

$\gamma d(T)$ = the present value cost (at time zero) of introducing the backstop at time T.

The analysis is assisted by use of the following conventional assumptions on the benefit and demand functions, and on the development cost function d(T).

A1: $p'(x+y) < 0$

A2: $R(x+y)$ is strictly concave

A3: $U(x+y) = \int_0^{p(x+y)} p(z)dz$

A4: $\bar{p} < p(0)$; $\bar{p}/(\bar{x}p'(\bar{x})) < -1$

A5: $d'(T) < 0$; $d''(T) > 0$; $\gamma \geqslant 0$. [6]

Assumptions A1-A3 require no comment. A4 implies, first, that the backstop technology will eventually be utilized. The second part of A4, that demand be at least unit elastic at the backstop supply price, insures that the cartel will supply the resource at prices below $\bar{p}$ if the initial stock of the resource is sufficiently large. A5 determines the nature of the time-cost trade-off in the development activity, and the resulting (present value) development cost function is illustrated in Figure 1.

2.2 The Cartel's Response Function

The first problem to consider is the optimal response of the cartel to a given introduction time for the substitute. Let $x(t/T)$ and $S(t/T)$ by the cartel output and resource stock along an optimal output path which is contingent upon the substitute's introduction time T. Then, $x(t/T)$ is the solution to:

$$\text{(7)} \qquad \underset{\{x(t)\}}{\text{maximize}} \int_0^T e^{-rt}(R(x(t))dt + e^{-rt}V(S(T))$$

$$\text{subject to:} \quad \int_0^T x(t)dt \leq S(0)$$

where $V(S(T))$ is the maximized present value of the stock $S(T)$ remaining at time T.

The solutions to (7) for extreme values of T are well understood. In particular, when $T = \infty$, the cartel is effectively unconstrained by the introduction to the substitute. The solution in this case is characterized by marginal revenue rising at rate r until extraction decreases to zero at the same instant that the resource is exhausted.[7] When $T = 0$, the substitute is introduced the instant that the market price exceeds the supply price of the substitute. Gilbert and Goldman (1978), Hoel (1978), and

Salant (1979) show that the solution requires that current marginal revenue rise at rate r until $\bar{p}$ is reached. Beyond that point, the cartel supplies the market demand at the substitute supply price, or just slightly below, until exhaustion of the stock.[8] Subsequent to exhaustion of conventional supplies, the backstop technology is utilized.

The solution for intermediate values of T is characterized below.

Proposition 2.1: Given A1, A2, A4 and A5, there exist times T_1, T_2, and T_3 satisfying $0 < T_1 < T_2 < T_3 \leq \infty$ such that

1. For $T \leq T_1$

(a) $x(t/T) = x(t/0)$; — $x(t/T)$ corresponds identically to optimal cartel output when the substitute is ready immediately.

(b) T_1 is defined by $x(T_1/0) = \bar{x}$

2. For $T_1 < T < T_2$

(a) $\dot{R}'/R' = r$ for $t < T$; — current marginal revenue rises at rate r prior to T.

(b) $x(T/T) < \bar{x}$ and $p(T/T) > \bar{p}$; — price rises above $\bar{p}$ prior to the substitute's introduction.

(c) $x(t/T) = \bar{x}$ and $p(t/T) = \bar{p}$ for $t > T$ until $S(t/T) = 0$; — after the substitute is introduced, the cartel produces at the rate $\bar{x}$ until the reserves are exhausted.

(d) $d(x(t/T))/dT > 0$ for $t \leq T$; — current cartel extraction increases with delays in the introduction time of the substitute.

3. For $T_2 \leq T < T_3$

(a) $R'/R' = r$ for $t < T$; — current marginal revenue rises at rate r prior to T.

(b) $x(T/T) < \bar{x}$, and $p(T/T) > \bar{p}$; — price rises above $\bar{p}$ prior to the substitute coming on line.

(c) $S(T/T) = 0$; — the cartel reserves are exhausted at the time the substitute is introduced.

(d) $d(x(t/T))/dT < 0$; — current cartel extraction decreases with delays in the introduction time of the substitute.

4. For $T \geq T_3$

(a) $x(t/T) = x(t/\infty)$; — $x(t/T)$ corresponds identically to optimal cartel output when the substitute is never introduced.

(b) T_3 is defined by $x(T_3/\infty) = 0$

The proof to Proposition 2.1 is relegated to Appendix 1. According to the proposition, the reaction of the cartel to the introduction of the substitute depends upon the interval in which T is contained. Representative price paths for the possible responses are illustrated in Figures 2a-2c. The solid curves represent the price path that prevails while total demand is satisfied by the resource; the dotted line refers to production from the substitute.

For the interval $T \varepsilon [0,T_1)$, changes in the time of introduction of the backstop have no effect on the cartel, because optimal price paths for

the cartel never reach the backstop price prior to T. According to part 2 of the Proposition, for $T \varepsilon (T_1,T_2)$, the market price of the resource (depicted in Fig. 2b), first rises above, and then falls discontinuously to the backstop price at the time when the substitute is introduced. In this case, T is not long enough for the cartel to exhaust its stock before entry occurs. Thus, after T, the cartel continues to produce at $\bar{x}$ until its stock is exhausted. As T increases, the response of the cartel is to increase its rate of extraction. The rate of production is maximized at $T=T_2$.

For $T \geq T_2$, the introduction of the substitute is sufficiently postponed such that the resource is exhausted by T (see Fig. 2c). As in the preceding case, the market price rises above the backstop price prior to T. A delay in the introduction of the substitute allows the cartel more time to exhaust the resource and thus decreases its rate of production. When T reaches T_3, the cartel is effectively unconstrained by the threat of entry.

The effect of T on the cartel's optimal pricing strategy in the initial period is illustrated in Figure 3. The Gilbert and Goldman (1978) and Hoel (1978) result that the initial price of the resource will be higher when the technology is available immediately than in the effective absence of a competitive substitute $(T \geq T_3)$ is obtained as a special case of Proposition 2.1. Our extended analysis indicates the existence of a time $\hat{T}$, such that if the introduction date lies in the interval $T \varepsilon [0,\hat{T})$, the cartel reacts to the pending threat of entry by increasing the initial price from the unconstrained level. However, the direction of the price response reverses when the technology becomes available in the interval $(\hat{T},T_3)$.

The <u>entire</u> price path faced by consumers shifts down from that path realized if no backstop were developed. At T_2 the maximum initial extraction rate chosen by the cartel is attained.

2.3 The Optimal Stackelberg Strategy for Consuming Nations

Given the response by the cartel to research strategies indexed by T, the consuming nations choose the optimal date of introduction that maximizes discounted consumer surplus, net of development costs. According to Proposition 2.1, $x(t/T) = x(t/0)$ for all $T \leq T_1$. Since the present value costs of research and development decrease with T, consumers will want to choose an introduction date $T \geq T_1$. Moreover, development costs are assumed to be such that the cartel is effectively constrained by the consumer's choice of introduction date for the substitute ($T < T_3$).

The optimal T is characterized as the solution to

$$\text{(9)} \qquad \underset{T\varepsilon(T_1,T_3)}{\text{Maximize}} \; W(T,\gamma,\bar{p},S(0)) = \underset{T\varepsilon(T_1,T_3)}{\text{maximize}} \int_0^T e^{-rt} N(x(t/T))dt + e^{-rt}\frac{N(\bar{x})}{r} - \gamma d(T)$$

where $N(x) = U(x) - p(x)x$ is the flow of consumer surplus generated by a stream of consumption x purchased at price p(x). The first two terms in (9) respectively represent the present value flow of surplus prior to and subsequent to the introduction date T. The third term is the present value cost of developing the substitute. The solution to the Stackelberg game is characterized in Proposition 2.2.

<u>Proposition 2.2:</u> Given A1-A5, $\gamma > 0$, and assuming a unique interior solution to (9), denoted by T^s, then T^s satisfies

(10) $$\int_0^T e^{-rt} N' \frac{\partial x}{\partial T} dt + e^{-rt} \; N(x(T/T)) - N(\bar{x}) \; - \gamma d'(T) = 0$$

for $T^S > T_1$ and $T \neq T_2$;

or

$W_1^-(T^S) \geq 0$, $W_1^+(T^S) \leq 0$ for $T^S = T_2$, where W_1^- and W_1^+ are the left and right derivatives of W with respect to T.[9]

(11) $$dT^S/d\gamma > 0, \; (> 0 \text{ for } T \neq T_2)$$

Equation (10) of Proposition 2.2 follows directly from differentiation of (9) with respect to T. According to 2c and 2d of Proposition 2.1, all the terms on the left hand side of (10) are positive when $T = T_1$; thus, T^S always strictly exceeds this value. Thus, the case studied by Gilbert and Goldman (1978) and Hoel (1978) never represents an optimal strategy for consuming nations capable of behaving strategically with respect to the development of substitutes. Some delay is always preferable, in order to realize lower initial prices (see Fig. 3) at the expense of prices realized later rising above the backstop supply price.

As the introduction date is postponed beyond T_1, the flow of surplus generated prior to the introduction of the backstop changes as described by the first term in (10). According to Proposition 2.1, this term is positive in the interval (T_1,T_2) and negative beyond T_2. Each instant the backstop is delayed beyond T_1, there is a loss in consumer surplus due to the market price exceeding the backstop supply price (see Fig. 2). This loss in surplus is captured by the second term in (10). Delaying introduction decreases development costs for all T; thus, the third term is always negative.

An interesting variant of the strategic problem is to consider the optimal strategy for consuming nations when $\gamma=0$, or development is costless (the substitute may be already available). It is still true that the solution to (9), T^S, exceeds T_1; but now the consuming nations have no vehicle by which they can credibly commit themselves to an introduction date later than T_1. If the cartel believed some announced research strategy $T' > T_1$ and followed its optimal extraction path $x(t/T')$, the consuming nations would introduce the backstop earlier than T', as soon as the market price exceeded the backstop supply price. Only an announced introduction date of T_1 or earlier is credible to the cartel, and thus T_1 is the only possible "perfect" equilibrium (in the sense of Selton (1975)) to the game. Under this equilibrium criterion, the results of Gilbert and Goldman (1978) and Hoel (1978) are obtained as a special case when the costs of research and development are zero. The consuming nations would be better off if they could undertake some development expenditure, even wastefully, so as to precommit themselves to an introduction date after T_1. This is an example of the benefits of strategic precommitment, which were investigated by Schelling (1960).

By allowing the parameter γ to vary, we can capture the effect of changing development costs on the optimal introduction date T^S. Equation (11) of Proposition 2.2 is obtained by totally differentiating the first order condition for T^S in eq. 10. Note that when $T^S = T_2$, a change in γ over a certain range has no effect on T^S. According to (11), the more costly the development of the substitute, the longer is the optimal delay in its introduction. In particular, when development costs are low, consuming nations may find it desirable to have the substitute technology

available prior to its utilization, in order to place an early lid on the cartel's prices. For sufficiently high development costs, the introduction date coincides with the date of exhaustion of the resource; there is no delay in utilization of the substitute.

3. Variations in the Substitute Price

3.1 Cartel Response

Cartel extraction may be influenced not only by T, but also by the price of the substitute. The response of the cartel to variations in the substitute price, denoted by p^s, is examined in this section. Here, government subsidies (taxes) on purchases of the substitute are introduced so that p^s may be less than (greater than) $\bar{p}$. We assume that governments of consuming nations can credibly alter the market price of the substitute, in order to affect desirable changes in cartel production. For example, the government might subsidize the consumption of oil substitutes by paying part of the installation costs of solar energy units. The effect of these actions on the behavior of resource cartels has been ignored in most public policy discussion.

To focus on the effects of prices, we assume that T is fixed, and that it has been determined by previous investment or technological constraints. Given T, the response of the cartel to variations in p^s is characterized in Proposition 3 below.

Proposition 3: Given the assumptions in Proposition 2.1, the resource cartel responds to a small change in the purchase price of a perfect substitute to the resource, to be developed at some time T, as follows. There exists a time $\bar{T} \varepsilon (T_1,T_2)$ such that $dx(t/T)/dp^s < 0$ for $T_1 < T < \bar{T}$ and

$t \leq T$; and $dx(t/T)/dp^s < 0$ for $\bar{T} < T < T_2$ and $t \leq T$. For $T = \bar{T}$ and $T > T_2$, the cartel is insensitive to small changes in p^s. At $T = T_2$, a small increase in p^s shifts the price path upwards; a decrease in p^s leaves the price path unchanged.

The proof of Proposition 3 is relegated to Appendix 2. For $T > T_2$ the cartel stops extracting just before T so that small changes in p^s do not affect production. For $T < T_2$ the cartel sells some of its stock at price p^s after T, and hence changes in p^s do affect production. An increase in p^s causes the rate of extraction to increase when T is sufficiently small; otherwise extraction decreases for increases in p^s.

3.2 Choice of the Substitute Price

We now envision that governments of consuming nations can cause at least small deviations in the substitute purchase price from the supply price to affect cartel production. For a given T and p^s, the discounted flow of consumer surplus is

$$(12) \qquad W(T, \gamma, \bar{p}, p^s, s(0)) = \int_0^T e^{-rt} N(x(t/T)dt$$

$$+ \frac{N(x(p^s))e^{-rT}[1-e^{-rZ}]}{r} + \frac{e^{-rZ}[V(x(p^s)) - \bar{p}\cdot x(p^s)]}{r}$$

where Z is the date of exhaustion of the resource. The first term on the RHS of (12) represents the consumer surplus flow during (0,T). (Note that the dependence of x(t/T) on p^s is suppressed in (12)) During (T,Z) output denoted by $x(p^s)$ equals market demand at the substitute purchase price, p^s, and the resultant flow of surplus is given by the second term in (12). The third term represents the flow of surplus after Z assuming that the tax or subsidy on the substitute remains after the cartel stops producing. This assumption does not affect our results.

By differentiating (12) with respect to p^s and evaluating it at $p^s = \bar{p}$ we can examine the effect of small taxes or subsidies on consumer welfare.

$$(13) \qquad \left.\frac{dW}{dp^s}\right|_{p^s=\bar{p}} = \int_0^T e^{-rt} N' \frac{dx(t/T)}{dp^s}\, dt + \frac{N' e^{-rT}\left[1 - e^{-r(Z-T)}\right]}{r} \frac{dx(\bar{p}^s)}{dp^s}$$

Using Proposition 3, equation (13) can be signed as follows,

$$(14) \qquad \left.\frac{dW}{dp^s}\right|_{p^s=\bar{p}} \begin{array}{lll} = 0 & & T > T_2;\ T = \bar{T} \\ < 0 & \text{for} & \bar{T} < T < T_2 \\ = ? & & T_1 < T < \bar{T} \end{array}$$

When $T > T_2$, cartel output is unaffected by marginal price changes and (at least small) taxes or subsidies are not warranted. For $\bar{T} < T < T_2$ the government can raise consumer welfare by subsidizing the substitute. A decrease in p^s causes extraction to increase prior to and after T. When $T_1 < T < \bar{T}$ variations in p^s have opposing effects on output prior to and after T, so that the welfare effects cannot be signed in general. Examples can be constructed in which increases in p^s may increase welfare since the loss in surplus from reduced output after T is more than compensated by the increase in the cartel rate of extraction prior to T.[10]

4. Conclusion

In this paper, the development and pricing of the backstop technology has been endogenized into the conventional exhaustible resource model. The market for the resource/substitute commodity mimics the "OPEC vs. the West" structure in that the cartel owner of the resource sells to a group of nations having the potential to develop a substitute to the exhaustible resource. In the case examined here the consumers are able to precommit

themselves to their strategy through the expenditure on the development of the substitute; the cartel, at best, determines its extraction path, given the development strategy.

The introduction date of the backstop technology has been shown to always occur after the resource price reaches the backstop supply or purchase price, when development of the substitute is costly. This suggests that attempting to time the development of new energy technologies, such as solar or synfuels, so as to introduce them when the price of conventional resources reaches their production cost, may not be an optimal strategy. The case commonly examined in the literature in which the technology is already available corresponds to the Stackelberg case when development costs are zero. In such a case, the backstop technology is introduced the instant at which the price reaches $\bar{p}$.

The Stackelberg model generalizes upon the previous papers examining a cartel's reaction to a backstop technology by considering the cartel's pricing behaviour for delayed introduction dates. In the case of an exogenously developed technology, the cartel owner increases the initial prices above the level it would set if unconstrained by potential entry (Gilbert and Goldman (1978), Hoel (1978), and Salant (1979)). In the general case a date is hown to exist such that the cartel lowers its initial prices from the unconstrained level in response to the introduction of a substitute after that date.

The latter result can be interpreted in another way. As introduction of the substitute is delayed between the times T_1 and T_2, the cartel sets lower initial prices; delays beyond T_2 cause the rate of extraction to become more conservative. Thus, up to time T_2 the consuming nations benefit

from an increase in surplus due to lower initial prices as well as the deferral in development costs. This indicates an intersting comparison with the case of a private R&D market, at least where one firm chooses the introduction date for the substitute, unthreatened by pre-emptive competition for the patent. Since the benefits of lower prices that accrue to consumers from delaying the technology are not appropriated by a private researcher, the individual's choice of introduction date may precede that chosen optimally by consumers. Such a result would run counter to that recently demonstrated for a non-exhaustible commodity by Dasgupta and Stiglitz (1981).

The date at which the consuming nations innovate has been shown to vary directly with development costs. If the costs of development are sufficiently small, then introduction of the technology may occur prior to its utilization (i.e., $T^s \epsilon (T_1, T_2)$). Such a strategy is optimal for low research costs since an early lid is placed on cartel prices, even though the technology remains idle over some interval. For large development costs the technology may not be introduced until the resource is depleted.

Possibilities for varying the purchase price of the substitute were examined in section 3. A complete analysis would involve consuming countries choosing T and p^s simultaneously to affect cartel production. Unfortunately, we are not able to obtain general analytical results for this model. However, the partial analysis in sections 2 and 3 has some implications for the development and pricing of resource substitutes. Propositions 2.2 and 3 suggest that the artificial pricing of the substitute is only important if the costs of development are small enough so that $T < T_2$. If development costs are moderate so that $T \epsilon (\bar{T}, T_2)$ then dropping the price

of the substitute causes the cartel to extract more rapidly which benefits consumers. For small development costs such that $T < \bar{T}$, consumers may benefit by raising the substitute price.

These conclusions about development and pricing policy require some qualification. First, they depend on the assumption that consuming nations can bind themselves to following long run policies as a means to influence cartel behaviour. The validity of this assumption is yet to be determined.

Another restrictive feature of our model is the open loop nature of the game; the decisions of both cartel and consuming nations are made at the beginning of the planning period. An important extension of this paper would be to allow R&D expenditures to be committed as a flow over the planning period, in a closed loop differential game, where the cartel is also able to revise its extraction plans based on the current state of technology in substitutes.

Third, our model abstracts from certain real world complexities. Perhaps the most important one is the uncertainty involved in finding a viable resource substitute. The incorporation of technological uncertainty in our analysis would be a difficult but interesting extension.

A final extension would be to make the <u>supply</u> price of the substitute endogenous. Thus, consuming nations may in fact be able to trade off an earlier introduction date against a higher production cost, and <u>vice versa</u>, for the same commitment of R&D expenditure. Section 3 provides a clue as to the results of such a model, in that raising the backstop supply price and delaying the introduction of the substitute have similar effects on the cartel's extraction policy. In both cases, the cartel may initially extract more rapidly, if the introduction of the substitute is sufficiently early.

Footnotes

* After this paper was completed, we learned that with the exception of Section 3, most of the results reported in this paper have been independently derived by Dasgupta, Gilbert and Stiglitz (1981).

1. During the Carter Administration, the Energy Security Corporation was set up to speed the development of synfuels, solar energy, and other oil substitutes. (See the Economic Report of the President (1980), pp. 112-116.) The Program has been dissolved under the Reagan Administration.

2. Non-economic methods for the cartel to effect the introduction of the substitutes are ruled out.

3. The analysis of other game theoretic situations where the cartel can partially commit itself to future supply decisions is contained in Lewis (1981).

4. For example, the capital costs of a 50,000 bbl/day demonstration plant for the coal liquefaction process is approximately \$1.7 billion; operating costs are \$240 million/yr. (Ralph Parsons (1978)).

5. This behaviour of the price path was recognized by Dasgupta, Gilbert and Stiglitz (1981).

6. This follows the characterization of the research and development costs contained in Dasgupta, Gilbert and Stiglitz (1981).

7. Implicit in the analysis is the assumption that the cartel cannot price discriminate or threaten not to supply the resource for certain time periods in order to earn high profits.

8. This is true provided that the stock is sufficiently large. (See Appendix 1).

9. When $T = T_2$, $x(t/T)$ is not differentiable with respect to T.

10. Consider this discrete time example. Assume that the demand curve is $p(x+y) = 1000-(x+y)$. Then consumer surplus is $(x+y)^2/2$. Suppose

the substitute becomes available at the beginning of the second time period, and that the cartel optimally exhausts after two periods. Proceeding as in the derivation of (13), we can show that the effect on consumer surplus of a small tax applied to the substitute is given by

$$\left.\frac{dW}{dp^s}\right|_{p^s=\bar{p}} = x(1/2)\frac{dx(1/2)}{dp^s} + Bx(\bar{p})\frac{dx(p^s)}{dp^s} = (B\bar{x} - x(p^s))\frac{dx(p^s)}{dp^s}$$

where $B = 1/(1+r)$ is the discount factor, and $x(1/2)$ is the output in period one given that the substitute is ready at the beginning of period two. Note that $Y = 0$ in period 2. Since the cartel only produces for two time periods, $dx(1/2)dp^s = -dx(p^s)/dp^s$. This is used to obtain the last equality in the equation above.

Suppose $\bar{p} = 850$, the cartel's initial stock is 275, and $B = .9$. The cartel maximizes present value profits by selling 125 units in period 1, and the entire market demand, 150 units, at the substitute supply price of 850 in period two as summarized in the table below. For this case, the equation above indicates $dW/dp^s = [(.9)(150) - 125]\, dx(p^s)/dp^s < 0$. Here consumers benefit by a substitute price subsidy. A drop in p^s causes output and consumer surplus to decrease initially, but this is more than offset by production and surplus increases in subsequent periods.

Period 1		Period 2	
Output	\$ 125	Output	\$ 150
Price	875	Price	850
Marginal Revenue	730	Discounted Price	765
		Discounted Marginal Revenue	630

Appendix 1

Proof of Proposition 2.1

Assume that the substitute comes on line at some time $T \geq 0$. Given an introduction date T, the cartel chooses an output path $x(t/T)$, and amount of resource to extract beyond T, $S(T/T)$, to

$$\text{(A1)} \qquad \underset{\{x(t)\}}{\text{maximize}} \int_0^T R(x(t))e^{-rt}dt + e^{-rt}V(S(T))$$

$$\text{subject to } S(0) = \int_0^T x(t)dt + S(T)$$

where

$$\text{(A2)} \qquad V(S(T)) = \underset{Z,\{x(t)\}}{\text{maximize}} = \int_T^Z R(x(t))e^{-r(t-T)}dt$$

$$\text{subject to } S(T) = \int_T^Z x(t)dt$$

$$\text{and } x(t) \geq \bar{x}$$

The last restriction in (A2) follows because it is optimal for the cartel to produce the entire output while it still has reserves.

Consider the problem in (A2) first. The solution to this control problem yields,

$$\text{(A3)} \qquad R'(x(t/T)) - \lambda \leq 0 \qquad (= \text{if } x > \bar{x})$$

$$\text{(A4)} \qquad \dot{\lambda} = r\lambda$$

$$\text{(A5)} \qquad \lim_{t \to Z} e^{-r(t-T)} [R(x(t/T) - \lambda x(t/T)] = 0$$

Since $x(t/T)$ is bounded away from zero and $S(T/T)$ is finite, $(Z-T)$ must also be finite. Hence, the bracketed term in (A5) is zero. This implies that $\lambda(Z) = p(Z) = \bar{p}$ by (A3), and the output path is given by

$$\text{(A6)} \qquad x(t/T) = \max [R'^{-1}(\lambda(T)e^{-r(T-t)}), \bar{x}]$$

where $\lambda(T) = \bar{p}e^{-r(Z-T)}$.

The resource constraint and (A6) implies that

$$\text{(A7)} \qquad \int_{T}^{Z} \max\,[R'^{-1}(\lambda(t)), \bar{x}]\,dt = S(T/T)$$

Changing the variable of integration from t to λ and noting that $\dot{\lambda} = r\lambda$, (A7) may be rewritten as

$$\text{(A8)} \qquad \int_{\lambda(T)}^{\bar{P}} \frac{\max\,[R'^{-1}(\lambda),\, \bar{x}]}{r\lambda}\,d\lambda = S(T/T)$$

Two things are apparent from (A8). First, $\lambda(T)$ must be decreasing with respect to $S(T/T)$. Second, there exists a maximum stock level, $\bar{S}$, such that if $S(T/T) \leq \bar{S}$, the rate of extraction beyond T equals $\bar{x}$ for $S(T/T) > \bar{S}$, output exceeds $\bar{x}$ for a while, then eventually declines to $\bar{x}$ as t approaches Z. $S(0)$ is assumed to exceed $\bar{S}$ throughout the analysis.

To determine the behaviour of the output path prior to T, the solution to the control problem in (A1) is required:

$$\text{(A9)} \qquad R'(x(t/T)) - \mu \leq 0 \qquad\qquad (= \text{ if } x > 0)$$

$$\text{(A10)} \qquad \dot{\mu} = r\mu$$

$$\text{(A11)} \qquad \mu(T) \geq V'(S(T/T)) \qquad\qquad (= \text{ if } S(T/T) > 0)$$

where $V'(S(T/T))$ is the shadow price of the resource, $\lambda(T)$, once the substitute is introduced. The proof of the proposition is divided into four parts corresponding to the four sections of the proposition.

1. $\underline{T \leq T_1}$

The extreme case of $T = 0$ has been investigated by Gilbert and Goldman (1978), Hoel (1978), and Salant (1979). They show that if $S(0) \geq \bar{S}$, $x(t/0)$ initially exceeds $\bar{x}$. At some time, T_1, the resource price reaches the backstop supply price and the resource is extracted at a constant rate $\bar{x}$ until exhaustion. T_1 is chosen optimally if

(A12) $\quad R'(\bar{x}) = \bar{p}e^{-r(Z-T_1)}$

Now consider the case when $0 < T \leq T_1$. First notice that the path $x(t/0)$ is optimal for all paths $x(t)$ satisfying $x(t) \geq \bar{x}$ for all t until the stock is exhausted. Then if $x(t/T) \neq x(t/0)$ it follows that $x(t/T) < \bar{x}$ for some $t \leq T$. By (A9) and (A10) $x(t/T)$ is monotonically declining so that $x(T/T) < \bar{x}$. Let $\lambda(t/T)$ be the shadow price of the resource at t for an introduction date T. Then since $x(t/T)$ lies below $x(t/0)$ it follows that $S(T/T) > S(T/0) > 0$. Hence, $R'(x(T/T)) = \mu(T) = \lambda(T/T)$ by (A11) and $\lambda(T/T) > \lambda(T/0)$ since $x(T/T) < x(T/0)$. But $S(T/T) > S(T/0)$ implies $\lambda(T/T) < \lambda(T/0)$ by (A8) thus providing a contradiction. This completes the proof to part 1 of Proposition 2.1

2. $\underline{T > T_1}$

Suppose at the introduction date, $T > T_1$, $x(T/T) \geq \bar{x}$. Then, $R'(x(T/T)) \leq R'(\bar{x})$ or according to (A9), $\mu(T) \leq \lambda(T_1/0)$. Since $T > T_1$, less of the resource is remaining at T if $x(T/T) \geq \bar{x}$ and $\lambda(T/T) > \lambda(T_1/0) \geq \mu(T)$ which violates (A11). Hence, $x(T/T) < \bar{x}$ and output falls below $\bar{x}$ and price rises above $\bar{p}$ prior to T, proving parts 2b and 3b of the proposition.

(A) $\underline{T_1 < T < T_2}$

Immediately after T, output, denoted by $x^+(T/T)$, must satisfy $x^+(T/T) \geq \bar{x}$. Suppose $x^+(T/T) > \bar{x}$. From (A6), $R'(x^+(T/T)) = \lambda(T)$. Then

(A13) $\quad \mu(T) = R'(x(T/T)) > R'(x^+(T/T)) = \lambda(T)$

(A13) implies that $S(T/T) = 0$ from (A11). Assuming some of the resource remains beyond T, $x^+(T/T) = \bar{x}$ as long as the cartel produces. This proves part 2c of the proposition.

Part 2d is proved below. Assuming $S(T/T) > 0$, (A6) implies that

(A14) $$\lambda(T) = e^{-rS(T/T)/\bar{x}_p}$$

and

(A15) $$S(T/T) = S(0) - \int_0^T x(t/T)dt$$

$$= S(0) - \int_0^T R'^{-1}[e^{-r(T-t)}\lambda(T)]dt$$

by (A2). Differentiating (A15) totally with respect to T and using (A15) yields,

(A16) $$\frac{dS(T/T)}{dT} = -\frac{\bar{x}[x(T/T) - r\int_0^T R''^{-1}(e^{-r(T-t)}\lambda(T))dt]}{[\bar{x} - r\int_0^T R''^{-1}(e^{-r(T-t)}\lambda(T))dt]} > -\bar{x}$$

since $x(T/T) < \bar{x}$. From (A6) and (A16) it follows that

(A17) $$\frac{dx(t/T)}{dT} = -rR''^{-1}e^{-r(T-t)}\lambda(T)\ [1 + \frac{dS(T/T)}{dT}\ \frac{1}{\bar{x}}] > 0$$

which completes the proof of part 2 of the proposition.

(B) $\underline{T_2 \leq T < T_3}$

Since $dS(T/T)/dT$ is negative and bounded away from zero by (A16) it follows that there is a time T_2 such that $S(T/T) = 0$ for $T \geq T_2$. The cartel output for $T > 0$ is given by

(A18) $$x(t/T) = R'^{-1}(\mu(T)e^{-r(T-t)}).$$

Therefore, the following holds for $T \geq T_2$,

(A19) $$S(0) = \int_0^T R'^{-1}[e^{-r(T-t)}\mu(T)]dt$$

Differentiating (A19) with respect to T yields

$$\text{(A12)} \qquad R'(\bar{x}) = \bar{p}e^{-r(Z-T_1)}$$

Now consider the case when $0 < T \leq T_1$. First notice that the path $x(t/0)$ is optimal for all paths $x(t)$ satisfying $x(t) \geq \bar{x}$ for all t until the stock is exhausted. Then if $x(t/T) \neq x(t/0)$ it follows that $x(t/T) < \bar{x}$ for some $t \leq T$. By (A9) and (A10) $x(t/T)$ is monotonically declining so that $x(T/T) < \bar{x}$. Let $\lambda(t/T)$ be the shadow price of the resource at t for an introduction date T. Then since $x(t/T)$ lies below $x(t/0)$ it follows that $S(T/T) > S(T/0) > 0$. Hence, $R'(x(T/T)) = \mu(T) = \lambda(T/T)$ by (A11) and $\lambda(T/T) > \lambda(T/0)$ since $x(T/T) < x(T/0)$. But $S(T/T) > S(T/0)$ implies $\lambda(T/T) < \lambda(T/0)$ by (A8) thus providing a contradiction. This completes the proof to part 1 of Proposition 2.1

2. $\underline{T > T_1}$

Suppose at the introduction date, $T > T_1$, $x(T/T) \geq \bar{x}$. Then, $R'(x(T/T)) \leq R'(\bar{x})$ or according to (A9), $\mu(T) \leq \lambda(T_1/0)$. Since $T > T_1$, less of the resource is remaining at T if $x(T/T) \geq \bar{x}$ and $\lambda(T/T) > \lambda(T_1/0) \geq \mu(T)$ which violates (A11). Hence, $x(T/T) < \bar{x}$ and output falls below $\bar{x}$ and price rises above $\bar{p}$ prior to T, proving parts 2b and 3b of the proposition.

(A) $\underline{T_1 < T < T_2}$

Immediately after T, output, denoted by $x^+(T/T)$, must satisfy $x^+(T/T) \geq \bar{x}$. Suppose $x^+(T/T) > \bar{x}$. From (A6), $R'(x^+(T/T)) = \lambda(T)$. Then

$$\text{(A13)} \qquad \mu(T) = R'(x(T/T)) > R'(x^+(T/T)) = \lambda(T)$$

(A13) implies that $S(T/T) = 0$ from (A11). Assuming some of the resource remains beyond T, $x^+(T/T) = \bar{x}$ as long as the cartel produces. This proves part 2c of the proposition.

Part 2d is proved below. Assuming $S(T/T) > 0$, (A6) implies that

(A14) $\lambda(T) = e^{-rS(T/T)/\bar{x}}\bar{p}$

and

(A15) $S(T/T) = S(0) - \int_0^T x(t/T)dt$

$$= S(0) - \int_0^T R'^{-1}[e^{-r(T-t)}\lambda(T)]dt$$

by (A2). Differentiating (A15) totally with respect to T and using (A15) yields,

(A16) $$\frac{dS(T/T)}{dT} = -\frac{\bar{x}\,[x(T/T) - r\int_0^T R''^{-1}(e^{-r(T-t)}\lambda(T))dt]}{[\bar{x} - r\int_0^T R''^{-1}(e^{-r(T-t)}\lambda(T))dt]} > -\bar{x}$$

since $x(T/T) < \bar{x}$. From (A6) and (A16) it follows that

(A17) $$\frac{dx(t/T)}{dT} = -rR''^{-1}e^{-r(T-t)}\lambda(T)\ \left[1 + \frac{dS(T/T)}{dT}\,\frac{1}{\bar{x}}\right] > 0$$

which completes the proof of part 2 of the proposition.

(B) $\underline{T_2 \leq T < T_3}$

Since $dS(T/T)/dT$ is negative and bounded away from zero by (A16) it follows that there is a time T_2 such that $S(T/T) = 0$ for $T \geq T_2$. The cartel output for $T > 0$ is given by

(A18) $x(t/T) = R'^{-1}(\mu(T)e^{-r(T-t)})$.

Therefore, the following holds for $T \geq T_2$,

(A19) $S(0) = \int_0^T R'^{-1}[e^{-r(T-t)}\mu(T)]dt$

Differentiating (A19) with respect to T yields

$$\text{(A20)} \qquad x(T/T) = -\int_0^T R''^{-1}\cdot\left[\frac{d}{dT}\, e^{-r(T-t)}\mu(T)\right] dt$$

If $x(T/T) > 0$ then (A20) implies that the bracketed term in the integral is positive. Using this information, the derivative of $x(t/T)$ with respect to T can be characterized by,

$$\text{(A21)} \qquad dx(t/T)/dT < 0 \qquad\qquad \text{if } x(T/T) > 0$$

This completes the proof of parts 3c and 3d of the proposition.

(C) $\underline{T \geq T_3}$

In this time interval the cartel is effectively unconstrained by entry. Gilbert and Goldman (1978) show that in this case, the cartel takes less time to extract a certain amount of the resource $S(0) - \bar{S}$ than when constrained by entry at $T = 0$. Thus, the initial rate of extraction is higher in the unconstrained than in the constrained case. The proof is not repeated here.

Appendix 2

Proof of Proposition 3

Along an equilibrium path for $T \epsilon (T_1, T_2)$,

(B1) $$R'(x(T/T)) = \lambda(T)$$

where R' is the marginal revenue and

(B2) $$\lambda(T) = p^s e^{-r(S(0) - \hat{S})/x^s}$$

where $\hat{S}$ is the amount of resource extracted prior to T. From (B1),

(B3) $$\frac{dx(T/T)}{dp^s} \equiv R''^{-1} \frac{d\lambda(T)}{dp^s}$$

Differentiating (B2) with respect to p^s gives

(B4) $$\frac{d\lambda(T)}{dp^s} = \lambda(T)\left[\frac{1}{p^s} + \frac{r\varepsilon(x^s)}{p^s x^s}(S(0) - \hat{S}) + \frac{r}{x^s}\frac{d\hat{S}}{dp^s}\right]$$

where $\varepsilon(\bar{x})$ is the price elasticity of demand evaluated at $\bar{x}$.

The resource constraint is given by

(B5) $$\hat{S} = \int_0^T R'^{-1}[\lambda(T)e^{-r(T-t)}]dt$$

Differentiating (B5) with respect to p^s gives

(B6) $$\frac{d\hat{S}}{dp^s} = \frac{x^s\frac{\lambda(T)}{p^s}\left(1 + \frac{r\varepsilon(x^s)}{x^s}(S(0) - \hat{S})\right)\int_0^T R''^{-1}\cdot e^{-r(T-t)}dt}{x^s - r\lambda(T)\int_0^T R''^{-1}\cdot e^{-r(T-t)}dt}$$

Substituting (B6) into (B4),

(B7) $$\frac{d\lambda(T)}{dp^s} = \frac{x^s\frac{\lambda(T)}{p^s}\left(1 + \frac{r\varepsilon(x^s)}{x^s}(S(0) - \hat{S})\right)}{x^s - r\lambda(T)\int_0^T R''^{-1}\cdot e^{-r(T-t)}dt}$$

Finally, substituting (B7) into (B3),

(B8) $$\frac{dx(T/T)}{dp^S} = R''^{-1}\, \frac{x^S \lambda(T)}{p^S} \cdot \frac{(1 + \frac{r\varepsilon(x^S)}{x^S}\, (S(0) - \hat{S}))}{(x^S - r\lambda(T) \int_0^T R''^{-1} \cdot e^{-r(T-t)} dt}$$

Therefore $\frac{dx(T/T)}{dp^S} \lesseqgtr 0$ as $(1 + \frac{r\varepsilon(x^S)}{x^S}\, (S(0) - \hat{S})) \gtreqless 0$

or

(B9) $$\hat{S} \gtreqless S(0) + \frac{x^S}{r\varepsilon(x^S)}$$

To determine the sign of (B9), the expression for $\hat{S}$ at $T = T_1$ and $T = T_2$ are determined. Consider $T = T_1$ first. The expression in (B2) can be rewritten as

(B10) $$p^S\, [1 + \frac{x^S}{\varepsilon(x^S)}] = p^S e^{-r(S(0) - \hat{S})/x^S}$$

or

(B11) $$\hat{S}(T_1) = S(0) + \frac{x^S \log[1 + \frac{1}{\varepsilon(x^S)}]}{r}$$

Since $\varepsilon(x^S) < -1$ by assumption (A4), $\log\,(1 + \frac{1}{\varepsilon(x^S)}) < \frac{1}{\varepsilon(x^S)}$ and by (B9)

(B12) $$\hat{S}(T_1) < S(0) + \frac{x^S}{r\varepsilon(x^S)}$$

Now consider $T = T_2$. At T_2, $S(T_2) = S(0)$ implying by (B9),

(B13) $$\hat{S}(T_2) > S(0) + \frac{x^S}{r\varepsilon(x^S)}$$

Since $\frac{d\hat{S}}{dT} > 0$ in the interval (T_1, T_2) according to (A16) in Appendix 1, there exists a $\bar{T}$ such that

(B14) $$\hat{S}(\bar{T}) = S(0) + \frac{x^S}{r\varepsilon(x^S)}$$

For $T \lesseqgtr \bar{T}$, $\frac{dx(T/T)}{dp^S} \gtrless 0$, if $T \varepsilon (T_1, T_2)$.

At $T = T_2$, the right hand derivative of $x(t/T)$ with respect to p^S is given by the expression in (B8). From (B13), an increase in p^S decreases the rate of extraction prior to T_2. A small decrease in p^S, however, changes the transversality condition evaluated at T_2 to,

(B15) $\quad R'(x(T/T)) > p^S$

and therefore, does not affect the cartel's extraction path. Similarly, for $T > T_2$, the cartel's extraction policy is insensitive to changes in p^S since the condition in (B15) is preserved. This completes the proof of Proposition 3.

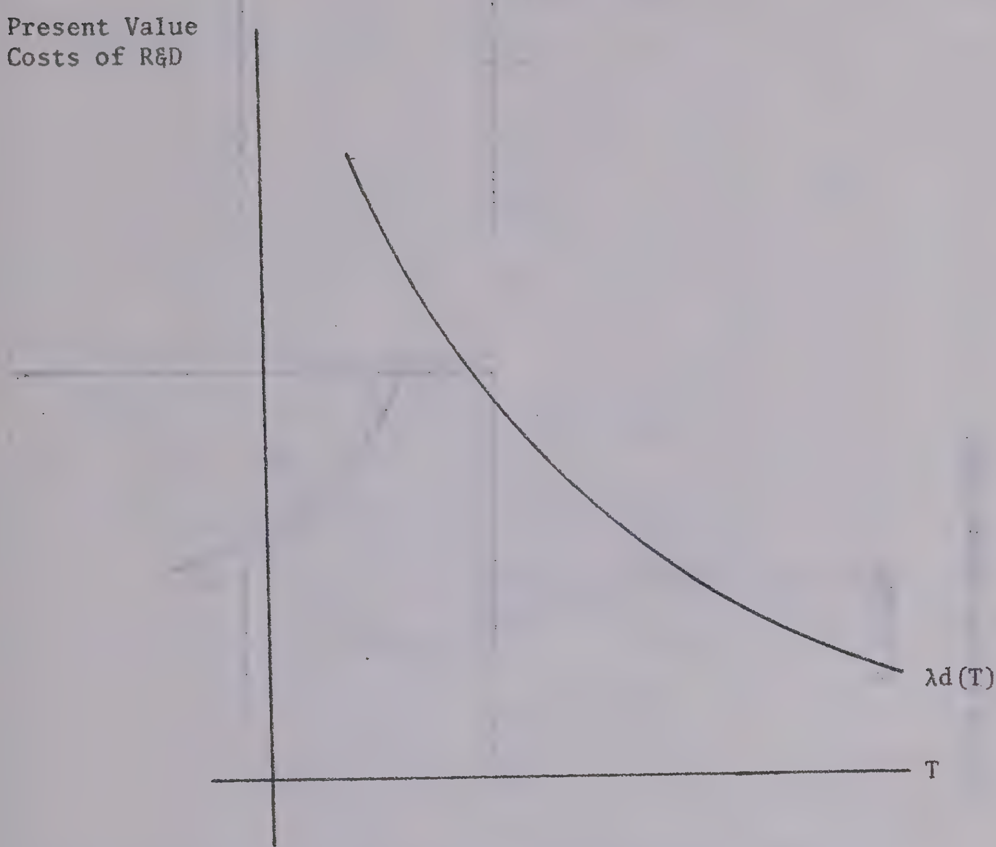
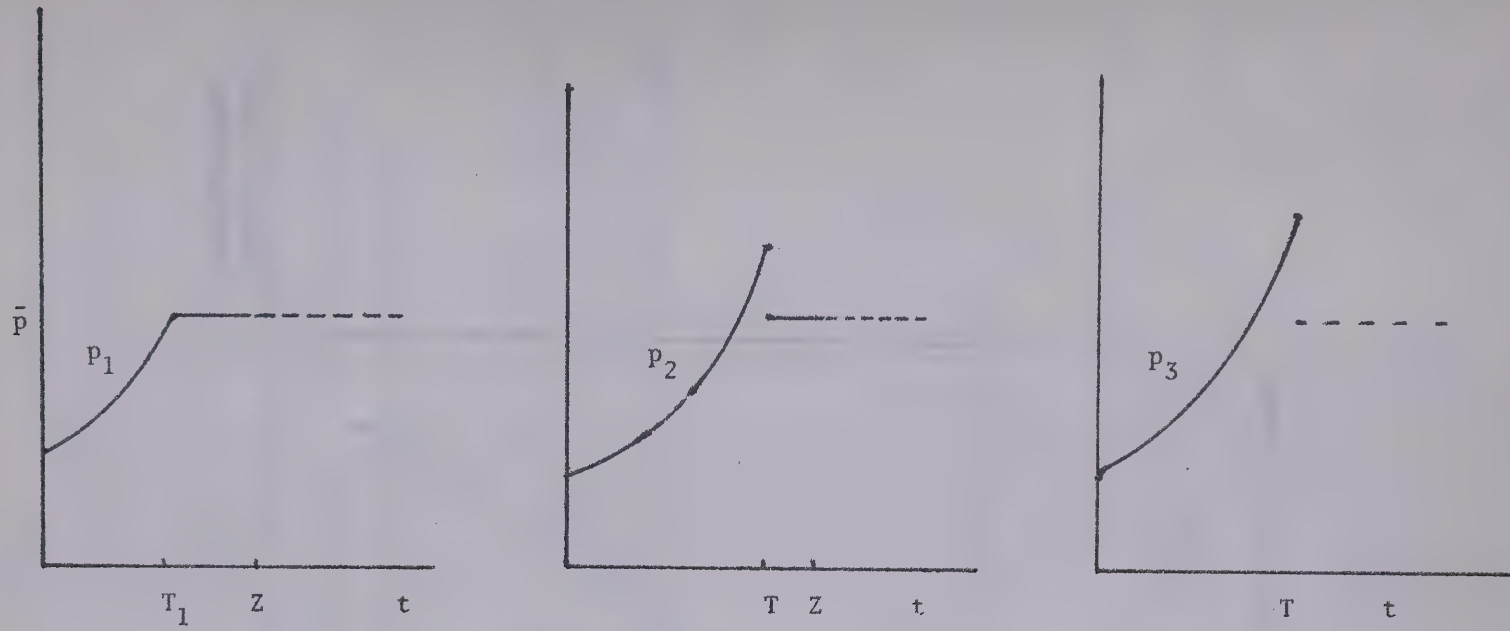


Figure 1

Research Cost Function



a. $T \leq T_1$ b. $T_1 < T < T_2$ c. $T > T_2$

Figure 2

Equilibrium Prices p(t/T)

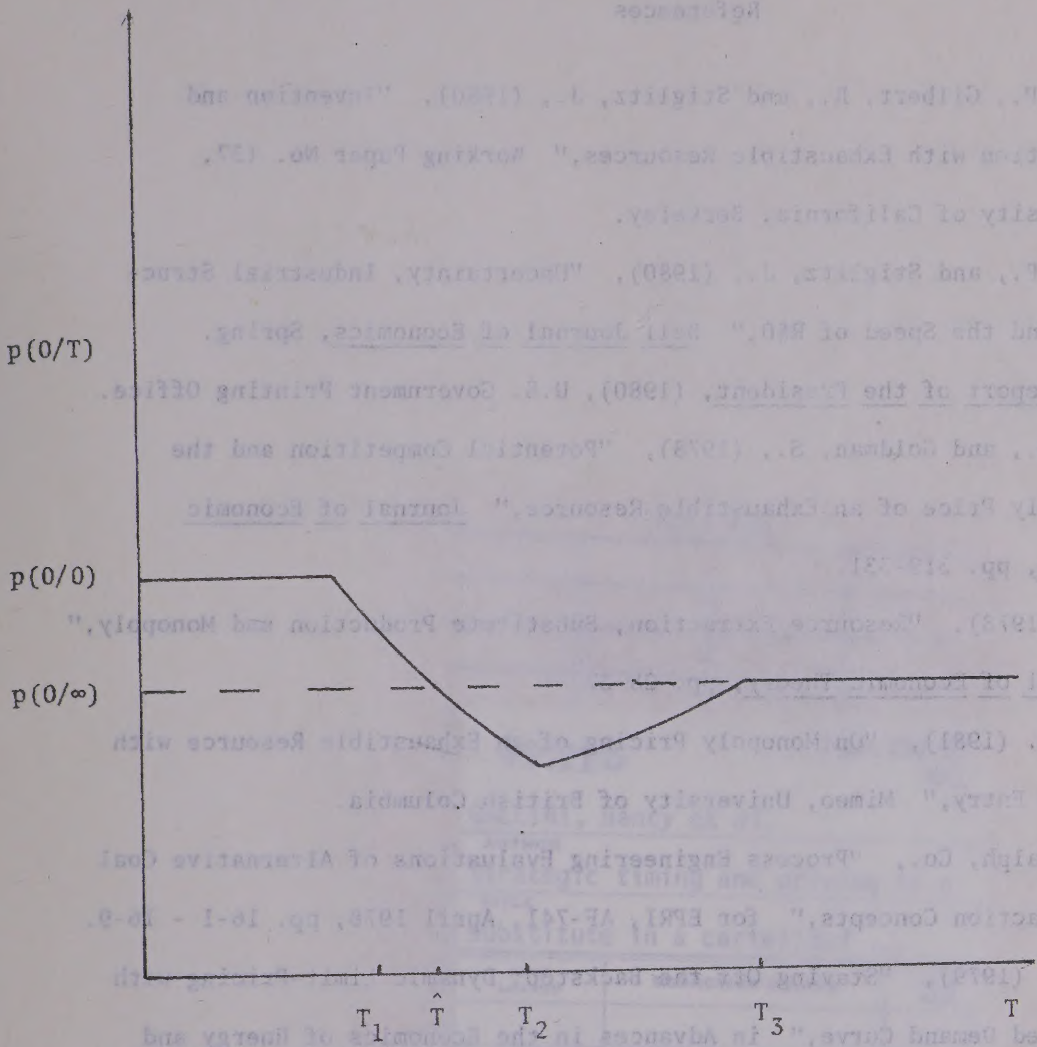


Figure 3

Cartel Reaction Function: p(0/T)

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